

Corrigé STATIQUE : Modélisation des actions mécaniques
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Exercice 1

Torseur résultant

$$\{\mathcal{T}_{FA}\} = \left\{ \begin{array}{c} \vec{F}_A \\ \vec{M}_{FA}(O) \end{array} \right\}_o = \left\{ \begin{array}{c} 100.\vec{x} \\ -200.\vec{z} \end{array} \right\}_o$$

$$\{\mathcal{T}_{FB}\} = \left\{ \begin{array}{c} \vec{F}_B \\ \vec{M}_{FB}(O) \end{array} \right\}_o = \left\{ \begin{array}{c} -50.\vec{x} + 100.\vec{y} \\ 250.\vec{z} \end{array} \right\}_o$$

$$\{\mathcal{T}_{FA+FB}\} = \left\{ \begin{array}{c} \vec{F} \\ \vec{M}(O) \end{array} \right\}_o = \left\{ \begin{array}{c} 50.\vec{x} + 100.\vec{y} \\ 50.\vec{z} \end{array} \right\}_o$$

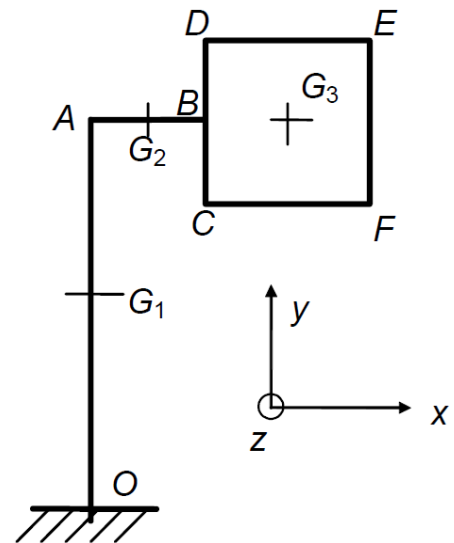
Support : Droite d'équation $y = 2.x - 1$

Exercice 2

Panneau indicateur

$$\{\mathcal{T}_{P1}\} = \left\{ \begin{array}{c} -3000.\vec{y} \\ \vec{0} \end{array} \right\}_o$$

$$\{\mathcal{T}_{P2}\} = \left\{ \begin{array}{c} -1000.\vec{y} \\ -1500.\vec{z} \end{array} \right\}_o$$

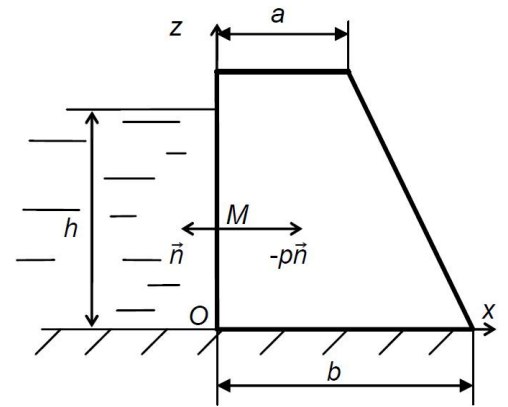


$$\{\mathcal{T}_{P3}\} = \left\{ \begin{array}{c} -4000.\vec{y} \\ -20000.\vec{z} \end{array} \right\}_o \quad \{\mathcal{T}_{Vent}\} = \left\{ \begin{array}{c} -6000.\vec{z} \\ -45000.\vec{x} + 30000.\vec{y} \end{array} \right\}_o$$

$$\{\mathcal{T}\} = \left\{ \begin{array}{c} -8000.\vec{y} - 6000.\vec{z} \\ -45000.\vec{x} + 30000.\vec{y} - 21500.\vec{z} \end{array} \right\}_o$$

Exercice 3

Barrage



Force élémentaire :
$$d\vec{F} = p.dS.\vec{x} = \rho.g.(h-z).l.dz.\vec{x}$$

On intègre et on trouve :
$$\vec{F} = \rho.g.l.\frac{h^2}{2}.\vec{x}$$

Remarque : $F = \text{Surface} \cdot \text{pression moyenne}$

Moment élémentaire en O :
$$d\vec{M}(O) = d\vec{M}(M) + \overrightarrow{OM} \wedge d\vec{F}$$

$$d\vec{M}(O) = z.\vec{z} \wedge \rho.g.(h-z).l.dz.\vec{x} = \rho.g.(h.z - z^2).l.dz.\vec{y}$$

On intègre et on trouve :
$$\vec{M}(O) = \rho.g.l.\frac{h^3}{6}.\vec{y}$$

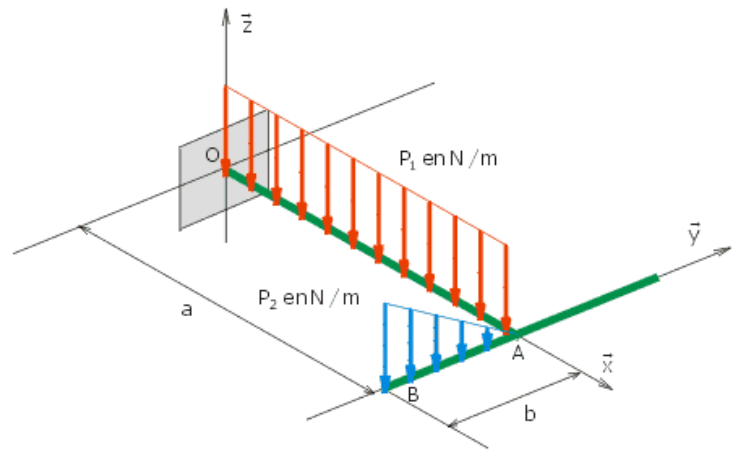
Centre de poussée : on cherche un point A tel que $\vec{M}(A) = \vec{0}$

$$\vec{M}(A) = \vec{M}(O) + \overrightarrow{AO} \wedge \vec{F} = \vec{0} \quad \Leftrightarrow \quad \dots \quad \Leftrightarrow \quad \overrightarrow{OA} = \frac{h}{3}.\vec{x}$$

Résultat classique d'une « répartition triangulaire »

Exercice 4

Poutre



$$\left\{ \mathcal{T}_{P_1} \rightarrow Poutre \right\} = \left\{ \begin{array}{c} \vec{F}_1 \\ \vec{M}_1(O) \end{array} \right\}_O = \left\{ \begin{array}{c} -P_1 \cdot a \cdot \vec{z} \\ +\frac{P_1 \cdot a}{2} \cdot \vec{y} \end{array} \right\}_O$$

$$\left\{ \mathcal{T}_{P_2} \rightarrow Poutre \right\} = \left\{ \begin{array}{c} \vec{F}_2 \\ \vec{M}_2(M) \end{array} \right\}_M = \left\{ \begin{array}{c} -\frac{P_2 \cdot b}{2} \cdot \vec{z} \\ \vec{0} \end{array} \right\}_M$$

M situé à 1/3 2/3 de AB

$$\vec{M}_2(O) = \vec{M}_2(M) + \overrightarrow{OM} \wedge \vec{F}_2 = \vec{0} + (a \cdot \vec{x} - \frac{2}{3} \cdot b \cdot \vec{y}) \wedge -\frac{P_2 \cdot b}{2} \cdot \vec{z}$$

$$\vec{M}_2(O) = \frac{P_2}{2} \cdot (a \cdot b \cdot \vec{y} + \frac{2}{3} \cdot b^2 \cdot \vec{x})$$

$$\left\{ \mathcal{T}_{P_2} \rightarrow Poutre \right\} = \left\{ \begin{array}{c} \vec{F}_2 \\ \vec{M}_2(O) \end{array} \right\}_O = \left\{ \begin{array}{c} -\frac{P_2 \cdot b}{2} \cdot \vec{z} \\ \frac{P_2}{2} \cdot (a \cdot b \cdot \vec{y} + \frac{2}{3} \cdot b^2 \cdot \vec{x}) \end{array} \right\}_O$$

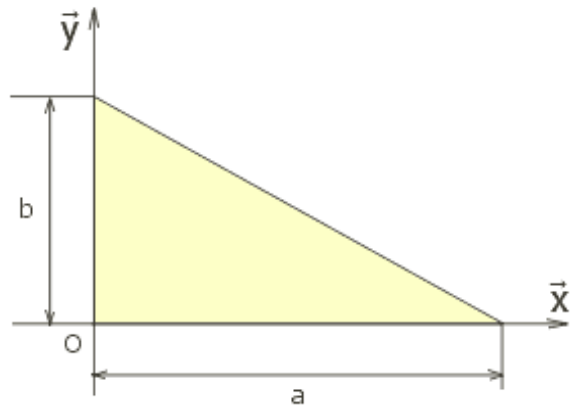
$$\left\{ \mathcal{T}_P \rightarrow Poutre \right\} = \left\{ \begin{array}{c} \vec{F} \\ \vec{M}(O) \end{array} \right\}_O = \left\{ \begin{array}{c} \vec{F}_1 + \vec{F}_2 \\ \vec{M}_1(O) + \vec{M}_2(O) \end{array} \right\}_O$$

Exercice 5

Triangle rectangle.

Intuitivement on peut dire :

$$\overrightarrow{OG} = \frac{1}{3} \cdot (a \cdot \vec{x} + b \cdot \vec{y})$$



Par calcul :
$$\overrightarrow{OG} = \frac{1}{V} \int_V \overrightarrow{OM} \cdot dv$$

$$x_G = \frac{1}{S} \int_S x \cdot ds = \frac{2}{a \cdot b} \int_0^a x \cdot \left(-\frac{b}{a} \cdot x + b\right) \cdot dx = \dots = \frac{a}{3}$$

$$\left\{ \mathcal{T}_{poids} \rightarrow S \right\} = \left\{ \begin{array}{c} \vec{F} \\ \vec{M}(G) \end{array} \right\}_G = \left\{ \begin{array}{c} -m \cdot g \cdot \vec{z} \\ \vec{0} \end{array} \right\}_G = \left\{ \begin{array}{c} \vec{F} \\ \vec{M}(O) \end{array} \right\}_O$$

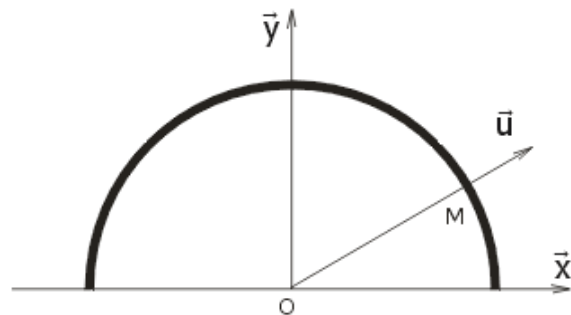
Directement :
$$\vec{M}(O) = -\frac{m \cdot g \cdot a}{3} \cdot \vec{z}$$

Ou par calcul :
$$\vec{M}(O) = \vec{M}(G) + \overrightarrow{OG} \wedge \vec{F}$$

Exercice 6

Demi-cercle.

$$\overrightarrow{OG} = \frac{1}{V} \int_V \overrightarrow{OM} \cdot dv$$



$$y_G = \frac{1}{L} \int_L \overrightarrow{OM} \cdot \vec{y} \cdot dl = \frac{1}{\pi \cdot r} \int_L r \cdot \vec{u} \cdot \vec{y} \cdot r \cdot d\theta$$

$$y_G = \frac{1}{\pi \cdot r} \int_0^\pi r \cdot \vec{u} \cdot \vec{y} \cdot r \cdot d\theta = \frac{r}{\pi} \int_0^\pi \sin \theta \cdot d\theta \qquad y_G = \frac{2 \cdot r}{\pi}$$