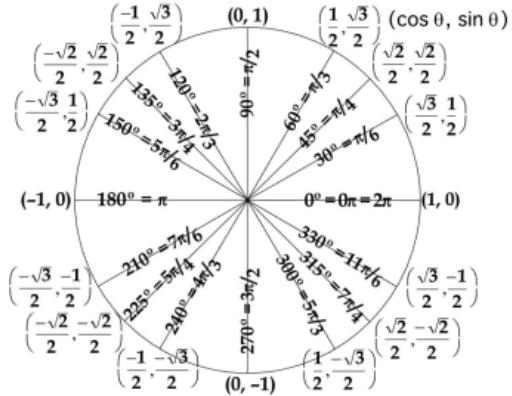


Calcul trigonométrique (Rappel)

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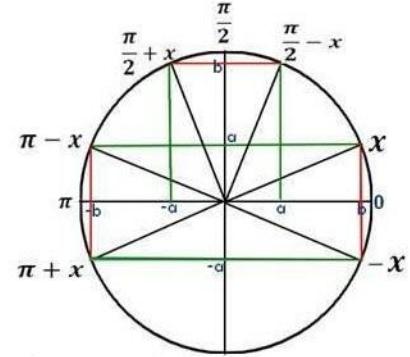
Tableau des valeurs habituelles et le cercle trigonométrique:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	



Relations entre les Ratios trigonométriques:

	$-x$	$\pi - x$	$\pi + x$	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$
\sin	$-\sin x$	$\sin x$	$-\sin x$	$\cos x$	$\cos x$
\cos	$\cos x$	$-\cos x$	$-\cos x$	$\sin x$	$-\sin x$



$$\cos(x + 2k\pi) = \cos x$$

$$\sin(x + 2k\pi) = \sin x$$

$$\tan(x + k\pi) = \tan x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \sin x \leq 1$$

$$\cos^2 x + \sin^2 x = 1$$

Équations trigonométriques essentielles:

$$\cos x = \cos a \Leftrightarrow x = a + 2k\pi \text{ ou } x = -a + 2k\pi$$

$$\sin x = \sin a \Leftrightarrow x = -a + 2k\pi \text{ ou } x = (\pi - a) + 2k\pi$$

$$\tan x = \tan a \Leftrightarrow x = a + k\pi ; (k \in \mathbb{Z})$$

Formules d'addition (soustraction):

$$\cos(a-b) = \cos a \times \cos b + \sin a \times \sin b$$

$$\sin(a-b) = \sin a \times \cos b - \cos a \times \sin b$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \times \tan b}$$

$$\cos(a+b) = \cos a \times \cos b - \sin a \times \sin b$$

$$\sin(a+b) = \sin a \times \cos b + \cos a \times \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \times \tan b}$$

Résultats:

En posant : $t = \tan \frac{a}{2}$

$$\sin a = \frac{2t}{1+t^2}$$

$$\cos a = \frac{1-t^2}{1+t^2}$$

$$\tan a = \frac{2t}{1-t^2}$$

$$\sin 2a = 2 \sin a \times \cos a$$

$$\tan 2a = \frac{2 \tan a}{1 - \tan^2 a}$$

$$\begin{aligned} \cos 2a &= \cos^2 a - \sin^2 a \\ &= 2 \cos^2 a - 1 \\ &= 1 - 2 \sin^2 a \\ &= \frac{1 - \tan^2 a}{1 + \tan^2 a} \end{aligned}$$

Linéarisation
(Formules de
CARNOT)

$$\sin^2 a = \frac{1 - \cos 2a}{2}$$

$$\cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\tan^2 a = \frac{1 - \cos 2a}{1 + \cos 2a}$$

Formules produit-somme:

$$\cos a \times \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \times \sin b = \frac{1}{2} [\cos(a+b) - \cos(a-b)]$$

$$\sin a \times \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\cos a \times \sin b = \frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

$$\cos p + \cos q = 2 \cos\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\cos p - \cos q = -2 \sin\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2 \sin\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$$

$$\sin p - \sin q = 2 \cos\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$$

$$\tan p + \tan q = \frac{\sin(p+q)}{\cos p \times \cos q}$$

$$\tan p - \tan q = \frac{\sin(p-q)}{\cos p \times \cos q}$$

Conversion de la formule $a \cos x + b \sin x$; $(a;b) \neq (0;0)$:

$$\begin{aligned} a \cos x + b \sin x &= \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \cos x + \frac{b}{\sqrt{a^2 + b^2}} \sin x \right) \\ &= \sqrt{a^2 + b^2} \cos(x - \alpha) \end{aligned}$$

Avec α un réel vérifiant : $\sin(\alpha) = \frac{b}{\sqrt{a^2 + b^2}}$ et $\cos(\alpha) = \frac{a}{\sqrt{a^2 + b^2}}$