

CORRECTION

Exercice 1

$$f(x) = \sin(2x + \pi)$$

Or $\sin(x + \pi) = -\sin x$ donc $\sin(2x + \pi) = -\sin 2x$

$$\text{Donc } f(x) = \sin(2x + \pi) = \boxed{-2 \sin x \cos x}$$

Exercice 2

$$\begin{aligned} f'(x) &= -\sin x \times \sin 2x + \cos x \times 2 \cos 2x - 2 \cos x \\ &= -\sin x \times 2 \sin x \cos x + \cos x \times 2 (2 \cos^2 x - 1) - 2 \cos x \\ &= -2 \sin^2 x \cos x + 2 \cos^3 x + 4 \cos^3 x - 4 \cos x \\ &= -2 (1 - \cos^2 x) \cos x + 4 \cos^3 x - 4 \cos x \\ &= 6 \cos^3 x - 6 \cos x \\ &= 6 \cos x (\cos^2 x - 1) \\ &= \boxed{6 \cos x (\cos x - 1)(\cos x + 1)} \end{aligned}$$

Exercice 3

$$f(x) = 2 \sin^2 x + 4 \sin x + 2$$

On pose $X = \sin x$

$$\text{Soit } f(x) = 2X^2 + 4X + 2$$

On résout $2X^2 + 4X + 2 = 0$

$\Delta = 0$ donc l'équation $2X^2 + 4X + 2 = 0$ admet une unique solution $x_0 = -1$

$$X = \sin x, \text{ d'où } \sin x = -1 \Leftrightarrow \boxed{x = -\frac{\pi}{2} + 2k\pi}$$

Exercice 4

$$1) -1 \leq \cos x \leq 1$$

$$\Leftrightarrow -1 \leq \cos\left(2x + \frac{\pi}{2}\right) \leq 1 \Leftrightarrow -1 \times 3 \leq 3 \times \cos\left(2x + \frac{\pi}{2}\right) \leq 1 \times 3$$

$$\Leftrightarrow \boxed{-3 \leq f(x) \leq 3}$$

$$2) f(-x) = 3 \cos\left(-2x + \frac{\pi}{2}\right)$$

Or $\cos x = \cos(-x)$

$$\text{Donc } f(-x) = 3 \cos\left(2x + \frac{\pi}{2}\right) = f(x)$$

La fonction f est donc paire.

$$\begin{aligned} 3) f(x + \pi) &= 3 \cos\left(-2(x + \pi) + \frac{\pi}{2}\right) = 3 \cos\left(-2x - 2\pi + \frac{\pi}{2}\right) \\ &= 3 \cos\left(-2x + \frac{\pi}{2}\right) = f(x). \end{aligned}$$

$$4) f'(x) = 3 \times 2 \times (-\sin\left(2x + \frac{\pi}{2}\right)) = -6 \sin\left(2x + \frac{\pi}{2}\right)$$

$$\begin{aligned} 5) \text{ a) Si } -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}, \text{ alors } -\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2} \Leftrightarrow 0 \leq 2x + \frac{\pi}{2} \leq \pi \\ \Leftrightarrow 0 \leq \sin\left(2x + \frac{\pi}{2}\right) \Leftrightarrow -6 \sin\left(2x + \frac{\pi}{2}\right) \leq 0 \Leftrightarrow f'(x) \leq 0 \end{aligned}$$

$$\begin{aligned} \text{Si } \frac{\pi}{4} \leq x \leq \frac{3\pi}{4}, \text{ alors } \frac{\pi}{2} \leq 2x \leq \frac{3\pi}{2} \Leftrightarrow \pi \leq 2x + \frac{\pi}{2} \leq 2\pi \\ \Leftrightarrow 0 \leq \sin\left(2x + \frac{\pi}{2}\right) \Leftrightarrow -6 \sin\left(2x + \frac{\pi}{2}\right) \geq 0 \Leftrightarrow f'(x) \geq 0 \end{aligned}$$

c)

x	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$
$f'(x)$	-	0	+
f	0	-3	3

$$6) y = f'\left(\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right) + f\left(\frac{\pi}{4}\right) \Leftrightarrow \boxed{y = -3}$$