

تمرين 1 : لنحسب بدلالة $\cos x$ و $\sin x$:

$$A = \sin\left(\frac{\pi}{4} + x\right) = \sin\left(\frac{\pi}{4}\right)\cos x + \cos\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$$

$$B = \cos\left(\frac{\pi}{4} + x\right) = \cos\left(\frac{\pi}{4}\right)\cos x - \sin\left(\frac{\pi}{4}\right)\sin x = \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$$

$$C = \sin\left(\frac{\pi}{6} - x\right) = \sin\left(\frac{\pi}{6}\right)\cos x - \cos\left(\frac{\pi}{6}\right)\sin x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$

$$D = \cos\left(\frac{\pi}{2} - x\right) = \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x = 0 + \sin x = \sin x$$

$$E = 2\cos\left(\frac{\pi}{6} - x\right) + \sqrt{2}\sin\left(\frac{\pi}{4} - x\right)$$

$$E = 2\left(\cos\left(\frac{\pi}{6}\right)\cos x + \sin\left(\frac{\pi}{6}\right)\sin x\right) + \sqrt{2}\left(\sin\left(\frac{\pi}{4}\right)\cos x - \cos\left(\frac{\pi}{4}\right)\sin x\right)$$

$$E = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right) + \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)$$

$$E = \sqrt{3}\cos x + \sin x + \cos x - \sin x$$

$$E = (\sqrt{3} + 1)\cos x$$

$$F = \sin(2x) - 3\cos\left(\frac{\pi}{6} + x\right)$$

$$F = 2\sin x \cos x - 3\left(\cos\left(\frac{\pi}{6}\right)\cos x - \sin\left(\frac{\pi}{6}\right)\sin x\right)$$

$$F = 2\sin x \cos x - 3\left(\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right)$$

$$F = 2\sin x \cos x - \frac{3\sqrt{3}}{2}\cos x + \frac{3}{2}\sin x$$

$$G = \cos\left(x + \frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3} - x\right) - \sin(x)$$

$$G = \cos\left(\frac{\pi}{3}\right)\cos x - \sin\left(\frac{\pi}{3}\right)\sin x - \left(\sin\left(\frac{\pi}{3}\right)\cos x - \cos\left(\frac{\pi}{3}\right)\sin x\right) - \sin x$$

$$G = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x - \sin x$$

$$G = \frac{1 - \sqrt{3}}{2}\cos x + \frac{-\sqrt{3} - 1}{2}\sin x$$

$$H = \cos x + \cos\left(x + \frac{2\pi}{3}\right) + \cos\left(x + \frac{4\pi}{3}\right)$$

$$H = \cos x + \cos(x)\cos\left(\frac{2\pi}{3}\right) - \sin x \sin\left(\frac{2\pi}{3}\right) + \cos x \cos\left(\frac{4\pi}{3}\right) - \sin x \sin\left(\frac{4\pi}{3}\right)$$

$$H = \cos x - \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = 0$$

$$\begin{cases} \cos(a+b) = \cos a \cos b - \sin a \sin b & \cos(a-b) = \cos a \cos b + \sin a \sin b \\ \sin(a+b) = \sin a \cos b + \cos a \sin b & \sin(a-b) = \sin a \cos b - \cos a \sin b \end{cases}$$

نذكر بالقواعد الأساسية:

$$\cos\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} ; \sin\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} ; \cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

نذكر بالقيم الخاصة:

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} ; \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} ; \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2} ; \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

تمرين 2 :

$$I = \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = \cos\left(\frac{\pi}{3}\right) \cos x + \sin\left(\frac{\pi}{3}\right) \sin x = \cos\left(\frac{\pi}{3} - x\right) = \cos\left(x - \frac{\pi}{3}\right)$$

$$J = \frac{\sqrt{2}}{2} \sin x + \frac{\sqrt{2}}{2} \cos x = \sin\left(\frac{\pi}{4}\right) \sin(x) + \cos\left(\frac{\pi}{4}\right) \cos x = \cos\left(\frac{\pi}{4}\right) \cos x + \sin\left(\frac{\pi}{4}\right) \sin(x) = \cos\left(x - \frac{\pi}{4}\right)$$

$$E = \frac{\sqrt{2}}{2} \sin x - \frac{\sqrt{2}}{2} \cos x = \cos\left(\frac{\pi}{4}\right) \sin x - \sin\left(\frac{\pi}{4}\right) \cos x = \sin\left(x - \frac{\pi}{4}\right)$$

$$F = \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \sin\left(\frac{\pi}{3}\right) \cos x + \cos\left(\frac{\pi}{3}\right) \sin x = \sin\left(x + \frac{\pi}{3}\right)$$

نطبق نفس القواعد السابقة لكن بشكل معكوس.

تمرين 3 :

$$G = \cos(2a) \cos a + \sin(2a) \sin a = \cos(2a - a) = \cos(a)$$

$$H = \cos(7a) \cos(3a) - \sin(7a) \sin(3a) = \cos(7a + 3a) = \cos(10a)$$

$$I = \frac{1}{2} \sin(3a) + \frac{\sqrt{3}}{2} \cos(3a) = \cos\left(\frac{\pi}{3}\right) \sin(3a) + \sin\left(\frac{\pi}{3}\right) \cos(3a) = \sin\left(3a + \frac{\pi}{3}\right)$$

$$J = \frac{\sqrt{2}}{2} \cos\left(\frac{a}{2}\right) + \frac{\sqrt{2}}{2} \sin\left(\frac{a}{2}\right) = \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{a}{2}\right) + \cos\left(\frac{\pi}{4}\right) \sin\left(\frac{a}{2}\right) = \sin\left(\frac{a}{2} + \frac{\pi}{4}\right) = \sin\left(\frac{2a + \pi}{4}\right)$$

تمرين 4 :

$$\cos\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right) = \left(\cos(x) \cos\left(\frac{\pi}{6}\right) - \sin(x) \sin\left(\frac{\pi}{6}\right)\right) \left(\cos(x) \cos\left(\frac{\pi}{6}\right) + \sin(x) \sin\left(\frac{\pi}{6}\right)\right)$$

$$= \left(\frac{\sqrt{3}}{2} \cos(x) - \frac{1}{2} \sin(x)\right) \left(\frac{\sqrt{3}}{2} \cos(x) + \frac{1}{2} \sin(x)\right)$$

$$= \left(\frac{\sqrt{3}}{2} \cos x\right)^2 - \left(\frac{1}{2} \sin x\right)^2 = \frac{3}{4} \cos^2 x - \frac{1}{4} \sin^2 x$$

$$= \frac{3}{4} \cos^2 x - \frac{1}{4} (1 - \cos^2 x) = \frac{3}{4} \cos^2 x - \frac{1}{4} + \frac{1}{4} \cos^2 x$$

$$\cos\left(x + \frac{\pi}{6}\right) \cos\left(x - \frac{\pi}{6}\right) = \cos^2 x - \frac{1}{4}$$

1

2

1

2

1

$$\begin{aligned}
(\sin x + \sin 5x)^2 + (\cos x + \cos 5x)^2 &= \sin^2 x + 2 \sin x \sin 5x + \sin^2 5x + \cos^2 x - 2 \cos x \cos 5x + \cos^2 5x \\
&= 1 + 1 + 2(\sin x \sin 5x - \cos x \cos 5x) \\
&= 2 - 2(\cos x \cos 5x - \sin x \sin 5x) \\
&= 2 - 2 \cos(5x - x) = 2 - 2 \cos(4x) \\
&= 2 - 2 \cos(2 \times 2x) = 2 - 2(2 \cos^2(2x) - 1) \\
&= 2 - 4 \cos^2(2x) + 2
\end{aligned}$$

2

$$(\sin x + \sin 5x)^2 + (\cos x + \cos 5x)^2 = 4 \cos^2(2x)$$

$$1 - \frac{\sqrt{2}}{2}(\cos 2x - \sin 2x) = 1 - \left(\frac{\sqrt{2}}{2} \cos 2x - \frac{\sqrt{2}}{2} \sin 2x \right) = 1 - \left(\cos\left(\frac{\pi}{4}\right) \cos(2x) - \sin\left(\frac{\pi}{4}\right) \sin(2x) \right)$$

$$1 - \frac{\sqrt{2}}{2}(\cos 2x - \sin 2x) = 1 - \cos\left(2x + \frac{\pi}{4}\right) = 1 - \cos\left(2\left(x + \frac{\pi}{8}\right)\right) = 2 \sin^2\left(x + \frac{\pi}{8}\right)$$

3

$$\cos(2x) = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x \Leftrightarrow \cos(2x) + 1 = 2 \cos^2 x \Leftrightarrow 1 - \cos(2x) = 2 \sin^2 x$$

$$\sin(2x) = 2 \sin x \cos x$$

نذكر بالخصيات: 🍋

