

CORRIGE – LA MERCI

EXERCICE 2B.1 Réduire les expressions :

$$A = 3\sqrt{2} + 5\sqrt{2} - 7\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$$

$$B = 5\sqrt{5} - 6\sqrt{3} - 8\sqrt{3} + \sqrt{5} = 6\sqrt{5} - 14\sqrt{3}$$

$$C = 3\sqrt{7} - 3\sqrt{5} - 5\sqrt{7} + 7\sqrt{5} = -2\sqrt{7} + 4\sqrt{5}$$

$$D = -4\sqrt{11} + 11\sqrt{13} + 13\sqrt{11} = 11\sqrt{13} + 9\sqrt{11}$$

EXERCICE 2B.2 Calculer les produits :

$$A = 7\sqrt{2} \times 3\sqrt{2} = 21 \times 2 = 42$$

$$B = 2\sqrt{5} \times 5\sqrt{7} = 10\sqrt{35}$$

$$C = 3\sqrt{2} \times 4\sqrt{5} = 12\sqrt{10}$$

$$D = 7\sqrt{3} \times (-2\sqrt{3}) = -14 \times 3 = -42$$

$$E = 5\sqrt{3} \times (-2\sqrt{5}) = -10\sqrt{15}$$

$$F = \sqrt{2} \times \sqrt{3} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{6}$$

EXERCICE 2B.3 Calculer les carrés :

$$A = (\sqrt{5})^2 = 5$$

$$B = (5\sqrt{2})^2 = 25 \times 2 = 50$$

$$C = (-2\sqrt{3})^2 = 4 \times 3 = 12$$

$$D = (2\sqrt{11})^2 = 4 \times 11 = 44$$

$$E = (6\sqrt{3})^2 = 36 \times 3 = 108$$

$$F = (3\sqrt{2})^2 = 9 \times 2 = 18$$

$$G = (-2\sqrt{7})^2 = 4 \times 7 = 28$$

$$H = (-9\sqrt{11})^2 = 81 \times 11 = 891$$

EXERCICE 2B.4 Écrire sous la forme « $a + b\sqrt{c}$ » (a , b et c sont des entiers relatifs) :

$$A = 2(3 + \sqrt{5}) = 6 + 2\sqrt{5}$$

$$B = 3(6 - \sqrt{2}) = 18 - 3\sqrt{2}$$

$$C = \sqrt{3}(4 + \sqrt{3}) = 4\sqrt{3} + 3$$

$$D = 2\sqrt{3}(5 - 2\sqrt{3}) = 10\sqrt{3} - 12$$

$$E = 5\sqrt{7}(-4 + 3\sqrt{7}) = -20\sqrt{7} + 105$$

$$F = -9\sqrt{11}(-2\sqrt{11} - 6) = 198 + 54\sqrt{11}$$

EXERCICE 2B.5 Écrire sous la forme « $a\sqrt{b}$ » (a et b sont des entiers relatifs, b est le plus petit possible) :

$$A = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$$

$$B = \sqrt{99} = \sqrt{9 \times 11} = 3\sqrt{11}$$

$$C = \sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$$

$$D = \sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$$

$$E = \sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$$

$$F = \sqrt{288} = \sqrt{144 \times 2} = 12\sqrt{2}$$

$$G = \sqrt{845} = \sqrt{169 \times 5} = 13\sqrt{5}$$

$$H = \sqrt{847} = \sqrt{121 \times 7} = 11\sqrt{7}$$

EXERCICE 2B.6 Écrire de la façon la plus simple possible :

$$A = \frac{\sqrt{2}}{2} - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} - \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$$

$$B = \frac{2}{\sqrt{3}} + \frac{3}{\sqrt{2}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} + \frac{3 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{2\sqrt{3}}{3} + \frac{3\sqrt{2}}{2}$$

$$C = \frac{4}{1 - \sqrt{2}} + \frac{3}{1 + \sqrt{2}} = \frac{4}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} + \frac{3}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = \frac{4(1 + \sqrt{2})}{1^2 - (\sqrt{2})^2} + \frac{3(1 - \sqrt{2})}{1^2 - (\sqrt{2})^2} = \frac{4 + 4\sqrt{2} + 3 - 3\sqrt{2}}{1 - 2}$$

$$= \frac{7 + \sqrt{2}}{-1} = -7 - \sqrt{2}$$

$$D = (1 + 2\sqrt{5})(2 - 5\sqrt{3}) = 1 - 5\sqrt{3} + 4\sqrt{5} - 10\sqrt{15}$$

$$E = (1 + 3\sqrt{2})(1 - 3\sqrt{2}) = 1^2 - (3\sqrt{2})^2 = 1 - 9 \times 2 = 1 - 18 = -17$$

$$F = (3 + 7\sqrt{2})(2\sqrt{2} - 11) = 6\sqrt{2} - 33 + 14 \times 2 - 77\sqrt{2} = -71\sqrt{2} - 5$$

$$G = 2\sqrt{7} + \sqrt{28} = 2\sqrt{7} + \sqrt{4} \times \sqrt{7} = 4\sqrt{7}$$

$$H = 4\sqrt{3} - \sqrt{48} = 4\sqrt{3} - \sqrt{16} \times \sqrt{3} = 0$$

$$I = 5\sqrt{2} + 3\sqrt{98} - 2\sqrt{242} = 5\sqrt{2} + 3\sqrt{49} \times \sqrt{2} - 2\sqrt{121} \times \sqrt{2} = 5\sqrt{2} + 21\sqrt{2} - 22 \times \sqrt{2} = 4\sqrt{2}$$

$$J = \sqrt{1 + \frac{3}{5}} \times \sqrt{1 - \frac{3}{5}} = \sqrt{\left(1 + \frac{3}{5}\right)\left(1 - \frac{3}{5}\right)} = \sqrt{1^2 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25}{25} - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$K = (\sqrt{7 - 2\sqrt{6}} + \sqrt{7 + 2\sqrt{6}})^2 = 7 - 2\sqrt{6} + 2\sqrt{7 - 2\sqrt{6}}\sqrt{7 + 2\sqrt{6}} + 7 + 2\sqrt{6} = 14 + 2\sqrt{(7 - 2\sqrt{6})(7 + 2\sqrt{6})}$$

$$= 14 + 2\sqrt{7^2 - (2\sqrt{6})^2} = 14 + 2\sqrt{49 - 4 \times 6} = 14 + 2\sqrt{25} = 14 + 10 = 24$$

$$L = \left(\sqrt{12-3\sqrt{7}} + \sqrt{12+3\sqrt{7}} \right)^2 = 12 - 3\sqrt{7} + 2\sqrt{12-3\sqrt{7}}\sqrt{12+3\sqrt{7}} + 12 + 3\sqrt{7} = 24 + 2\sqrt{(12-3\sqrt{7})(12+3\sqrt{7})}$$

$$= 24 + 2\sqrt{12^2 - (3\sqrt{7})^2} = 24 + 2\sqrt{144 - 9 \times 7} = 24 + 2\sqrt{144 - 63} = 24 + 2\sqrt{81} = 24 + 18 = 42$$

EXERCICE 2B.7 Ecrire sans racine au dénominateur :

$$A = \frac{3}{\sqrt{5}+1} = \frac{3}{\sqrt{5}+1} \times \frac{\sqrt{5}-1}{\sqrt{5}-1} = \frac{3(\sqrt{5}-1)}{(\sqrt{5})^2-1^2} = \frac{3\sqrt{5}-3}{5-1} = \frac{3\sqrt{5}-3}{4}$$

$$B = \frac{5}{1+\sqrt{2}} = \frac{5}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{5(1-\sqrt{2})}{1^2-(\sqrt{2})^2} = \frac{5-5\sqrt{2}}{1-2} = \frac{5-5\sqrt{2}}{-1} = 5\sqrt{2}-5$$

$$C = \frac{1+\sqrt{5}}{3-\sqrt{5}} = \frac{1+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(1+\sqrt{5})(3+\sqrt{5})}{3^2-(\sqrt{5})^2} = \frac{3+\sqrt{5}+3\sqrt{5}+5}{9-5} = \frac{8+4\sqrt{5}}{4} = 2+\sqrt{5}$$

$$D = \frac{1-\sqrt{5}}{3+\sqrt{5}} = \frac{1-\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(1-\sqrt{5})(3-\sqrt{5})}{3^2-(\sqrt{5})^2} = \frac{3-\sqrt{5}-3\sqrt{5}+5}{9-5} = \frac{8-4\sqrt{5}}{4} = 2-\sqrt{5}$$

$$E = \frac{1+\sqrt{7}}{2-\sqrt{7}} = \frac{1+\sqrt{7}}{2-\sqrt{7}} \times \frac{2+\sqrt{7}}{2+\sqrt{7}} = \frac{(1+\sqrt{7})(2+\sqrt{7})}{2^2-(\sqrt{7})^2} = \frac{2+\sqrt{7}+2\sqrt{7}+7}{4-7} = \frac{9+3\sqrt{7}}{-3} = -3-\sqrt{7}$$

$$F = \frac{7+\sqrt{2}}{5-\sqrt{3}} = \frac{7+\sqrt{2}}{5-\sqrt{3}} \times \frac{5+\sqrt{3}}{5+\sqrt{3}} = \frac{(7+\sqrt{2})(5+\sqrt{3})}{5^2-(\sqrt{3})^2} = \frac{35+7\sqrt{3}+5\sqrt{2}+\sqrt{6}}{25-3} = \frac{35+7\sqrt{3}+5\sqrt{2}+\sqrt{6}}{22}$$