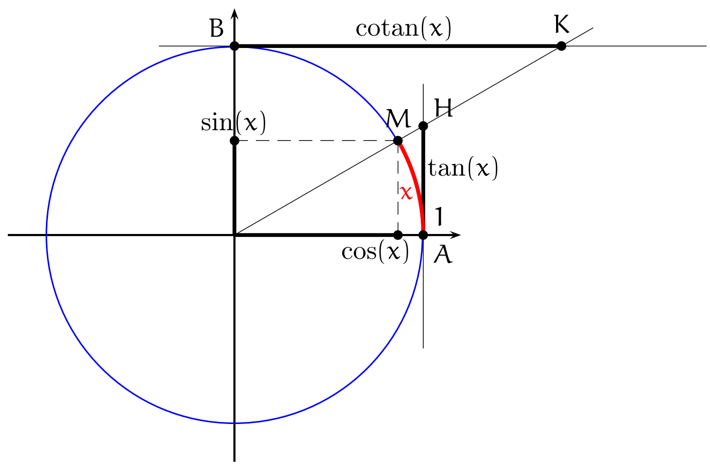
Formulaire de trigonométrie circulaire



$\cos(x) =$ abscisse de M

$\sin(x) =$ ordonnée de M

$\tan(x) = \overline{AH}$

$\text{cotan}(x) = \overline{BK}$

$e^{ix} = z_M$

Pour $x \notin \dfrac{\pi}{2} + \pi\mathbb{Z}$, $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$ et pour $x \notin \pi\mathbb{Z}$, $\text{cotan}(x) = \dfrac{\cos(x)}{\sin(x)}$. Enfin pour $x \notin \dfrac{\pi}{2}\mathbb{Z}$, $\text{cotan}(x) = \dfrac{1}{\tan(x)}$.

Valeurs usuelles.

x en °	0	30	45	60	90
x en rd	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$
$\sin(x)$	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0
$\tan(x)$	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\text{cotan}(x)$	∞	$\sqrt{3}$	1	$\dfrac{1}{\sqrt{3}}$	0

$$\forall x \in \mathbb{R},\ \cos^2 x + \sin^2 x = 1$$
$$\forall x \notin \frac{\pi}{2} + \pi\mathbb{Z},\ 1 + \tan^2 x = \frac{1}{\cos^2 x}.$$
$$\forall x \notin \pi\mathbb{Z},\ 1 + \text{cotan}^2 x = \frac{1}{\sin^2 x}.$$

addition d'un tour

$\cos(x + 2\pi) = \cos x$
$\sin(x + 2\pi) = \sin x$
$\tan(x + 2\pi) = \tan x$
$\text{cotan}(x + 2\pi) = \text{cotan}\, x$

addition d'un demi-tour

$\cos(x + \pi) = -\cos x$
$\sin(x + \pi) = -\sin x$
$\tan(x + \pi) = \tan x$
$\text{cotan}(x + \pi) = \text{cotan}\, x$

angle opposé

$\cos(-x) = \cos x$
$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$
$\text{cotan}(-x) = -\text{cotan}\, x$

angle supplémentaire

$\cos(\pi - x) = -\cos x$
$\sin(\pi - x) = \sin x$
$\tan(\pi - x) = -\tan x$
$\text{cotan}(\pi - x) = -\text{cotan}\, x$

angle complémentaire

$\cos(\dfrac{\pi}{2} - x) = \sin x$
$\sin(\dfrac{\pi}{2} - x) = \cos x$
$\tan(\dfrac{\pi}{2} - x) = \text{cotan}\, x$
$\text{cotan}(\dfrac{\pi}{2} - x) = \tan x$

quart de tour direct

$\cos(x + \dfrac{\pi}{2}) = -\sin x$
$\sin(x + \dfrac{\pi}{2}) = \cos x$
$\tan(x + \dfrac{\pi}{2}) = -\text{cotan}\, x$
$\text{cotan}(x + \dfrac{\pi}{2}) = -\tan x$

quart de tour indirect

$\cos(x - \dfrac{\pi}{2}) = \sin x$
$\sin(x - \dfrac{\pi}{2}) = -\cos x$
$\tan(x - \dfrac{\pi}{2}) = -\text{cotan}\, x$
$\text{cotan}(x - \dfrac{\pi}{2}) = -\tan x$

Formules d'addition

$\cos(a+b) = \cos a \cos b - \sin a \sin b$
$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\sin(a+b) = \sin a \cos b + \sin b \cos a$
$\sin(a-b) = \sin a \cos b - \sin b \cos a$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Formules de duplication

$\cos(2a) = \cos^2 a - \sin^2 a$
$= 2\cos^2 a - 1$
$= 1 - 2\sin^2 a$
$\sin(2a) = 2 \sin a \cos a$

$$\tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$$

Formules de linéarisation

$$\cos a \cos b = \frac{1}{2}(\cos(a-b) + \cos(a+b))$$
$$\sin a \sin b = \frac{1}{2}(\cos(a-b) - \cos(a+b))$$
$$\sin a \cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))$$
$$\cos^2 a = \frac{1+\cos(2a)}{2}$$
$$\sin^2 a = \frac{1-\cos(2a)}{2}$$

Formules de factorisation

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$
$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$$

cos x, sin x et tan x en fonction de t=tan(x/2)

$$\cos x = \frac{1-t^2}{1+t^2}$$
$$\sin x = \frac{2t}{1+t^2}$$
$$\tan x = \frac{2t}{1-t^2}$$

Divers

$$1 + \cos x = 2\cos^2\frac{x}{2}$$
$$1 - \cos x = 2\sin^2\frac{x}{2}$$
$$\cos(3x) = 4\cos^3 x - 3\cos x$$
$$\sin(3x) = 3\sin x - 4\sin^3 x$$

Résolution d'équations

$\cos x = \cos a \Leftrightarrow$
$\exists k \in \mathbb{Z}/\ x = a + 2k\pi$
ou
$\exists k \in \mathbb{Z}/\ x = -a + 2k\pi$

$\sin x = \sin a \Leftrightarrow$
$\exists k \in \mathbb{Z}/\ x = a + 2k\pi$
ou
$\exists k \in \mathbb{Z}/\ x = \pi - a + 2k\pi$

$\tan x = \tan a \Leftrightarrow$
$\exists k \in \mathbb{Z}/\ x = a + k\pi$

Exponentielle complexe

$\forall x \in \mathbb{R},\ e^{ix} = \cos x + i \sin x.$

Valeurs usuelles

$$e^0 = 1,\ e^{i\pi/2} = i,\ e^{i\pi} = -1,\ e^{-i\pi/2} = -i,\ e^{2i\pi/3} = j = -\frac{1}{2} + i\frac{\sqrt{3}}{2},\ \sqrt{2}e^{i\pi/4} = 1 + i.$$

Propriétés algébriques

$\forall x \in \mathbb{R},\ |e^{ix}| = 1.$

$$\forall (x,y) \in \mathbb{R}^2,\ e^{ix} \times e^{iy} = e^{i(x+y)},\quad \frac{e^{ix}}{e^{iy}} = e^{i(x-y)},\quad \frac{1}{e^{ix}} = e^{-ix} = \overline{e^{ix}}$$

Formules d'Euler

$$\forall x \in \mathbb{R},\ \cos x = \frac{e^{ix} + e^{-ix}}{2} \text{ et } e^{ix} + e^{-ix} = 2\cos x.$$
$$\forall x \in \mathbb{R},\ \sin x = \frac{e^{ix} - e^{-ix}}{2i} \text{ et } e^{ix} - e^{-ix} = 2i\sin x.$$

Formule de Moivre

$\forall x \in \mathbb{R},\ \forall n \in \mathbb{Z},\ (e^{ix})^n = e^{inx}.$