تحت إشراف : الأستاذ :
بوشعيب المانسي

# تصحيح الفرض 04

Groupe B

من إنجاز التلميذة صالحة جبوج

## التمرين 01 :

1)
$$u_{n+1} = \frac{2u_n}{2u_n+1}$$
$$u_{n+1} = \frac{2u_n+1-1}{2u_n+1}$$
$$u_{n+1} = 1 - \frac{1}{2u_n+1}$$

- نبين أن $\forall n\in\mathbb{N} \quad 0<u_n<\frac{1}{2}$

من أجل $n=0$ : $0<u_0=\frac{1}{5}<\frac{1}{2}$ (V)

نفترض أن $0<u_n<\frac{1}{2}$

ونبين أن $0<u_{n+1}<\frac{1}{2}$

$$0<u_n<\frac{1}{2} \Rightarrow 1<2u_n+1<2$$
$$\Rightarrow -1<\frac{-1}{2u_n+1}<-\frac{1}{2}$$
$$\Rightarrow 0<u_{n+1}<\frac{1}{2}$$

وعليه $\forall n\in\mathbb{N} \quad 0<u_{n+1}<\frac{1}{2}$

2)
$$u_{n+1}-u_n = \frac{2u_n-2u_n^2-u_n}{2u_n+1}$$
$$= \frac{u_n-2u_n^2}{2u_n+1}$$
$$= \frac{u_n(1-2u_n)}{2u_n+1}$$

* $0<1-2u_n<1$

و $0<2u_n+1<1$

إذن $u_{n+1}-u_n>0$

وعليه $u_n$ تزايدية.

3)
$$v_{n+1} = \frac{3^{n+1}u_{n+1}}{2u_{n+1}-1}$$
$$v_{n+1} = \frac{3^{n+1}\times\frac{2u_n}{2u_n+1}}{2\times\frac{2u_n}{2u_n+1}-1}$$
$$v_{n+1} = \frac{3^{n+1}\times 2u_n}{2u_n-1}$$
$$v_{n+1} = \frac{3^n\times 3\times 2u_n}{2u_n-1}$$
$$v_{n+1} = \frac{6\times 3^n u_n}{2u_n-1}$$
$$v_{n+1} = 6v_n$$

ومنه $v_n$ متتالية هندسية أساسها 6 .

$$v_n=v_0q^n \Leftrightarrow v_n = -\frac{1}{3}\times 6^n$$

ب- $v_n=\frac{3^n u_n}{2u_n-1} \Leftrightarrow v_n(2u_n-1)=3^n u_n$

$$\Leftrightarrow u_n(2v_n-3^n)=v_n$$
$$\Leftrightarrow u_n=\frac{v_n}{2v_n-3^n}$$
$$\Leftrightarrow u_n=\frac{-\frac{6^n}{3}}{-\frac{2\times 6^n}{3}-\frac{3^{n+1}}{3}}$$
$$\Leftrightarrow u_n=\frac{6^n}{2\times 6^n+3^{n+1}}$$
$$u_n=\frac{\cancel{3^n}\times 2^n}{\cancel{3^n}(2^{n+1}+3)}$$
$$u_n=\frac{2^n}{2^{n+1}+3}$$

$$\lim_{n\to+\infty} u_n = \lim_{n\to+\infty}\frac{\cancel{2^n}}{\cancel{2^n}\left(2+\frac{3}{2^n}\right)}$$
$$= \lim_{n\to+\infty}\frac{1}{2+\frac{3}{2^n}}$$
$$= \frac{1}{2}$$

" $2>1$ ; $\lim_{n\to+\infty} 2^n=+\infty$ ; $\lim_{n\to+\infty}\frac{3}{2^n}=0$ "

## التمرين 02:

①

$$\frac{b-a}{b-c} = \frac{5+i+1+i}{5+i-3-7i}$$

$$= \frac{6+2i}{2-6i}$$

$$= i$$

و $\left|\frac{b-a}{b-c}\right| = 1$.

$|b-a| = |b-c| \Leftrightarrow AB = BC$

$(\overrightarrow{CB}, \overrightarrow{AB}) = \frac{\pi}{2}\ [2\pi]$

إذن ABC مثلث متساوي الساقين وقائم الزاوية في B.

②

ABCD متوازي الأضلاع $\Leftrightarrow \overrightarrow{AB} = \overrightarrow{DC}$

$\Leftrightarrow aff(\overrightarrow{AB}) = aff(\overrightarrow{DC})$

$\Leftrightarrow b-a = c-d$

$\Leftrightarrow d = c-b+a$

$\Leftrightarrow d = 3+7i-5-i-1-i$

$d = 5i-3$

-③

$$\frac{c-a}{d-b} = \frac{3+7i+1+i}{5i-3-5-i}$$

$$= \frac{8i+4}{4i-8}$$

$$= \frac{2i+1}{i-2} = \frac{(1+2i)(-2-i)}{5}$$

$$= -\frac{5i}{5}$$

$$= -i$$

$\overline{(\overrightarrow{BD}\ ;\ \overrightarrow{AC})} = -\frac{\pi}{2}\ [2\pi]$ *

$\frac{|c-a|}{|d-b|} = 1$ &

أي $|c-a| = |d-b|$

$\overline{(\overrightarrow{CB}, \overrightarrow{AB})} = \frac{\pi}{2}[2\pi]$ و $AC = BD$.

إذن ABCD مربع.

## التمرين 03:

-①

$Z_1 = -\frac{3}{2} + \frac{\sqrt{3}}{2} i$.

$|Z_1| = \sqrt{\left(-\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{3}$.

$\cos\theta = \frac{-3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2\sqrt{3}} = \frac{1}{2}$

$\theta = \frac{5\pi}{6}$

$Z_1 = \left[\sqrt{3}\ ;\ \frac{5\pi}{6}\right]$

$Z_2 = 2-2i$

$|Z_2| = \sqrt{4+4}$

$= 2\sqrt{2}$

$\cos\theta = \frac{2}{2\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = -\frac{2}{2\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\theta = -\frac{\pi}{4}$

$Z_2 = \left[2\sqrt{2}\ ;\ -\frac{\pi}{4}\right]$

حساب: $\lim_{x\to+\infty} x+1-\ln(x+3)$

$$\lim_{x\to+\infty} x+1-\ln(x+3) = \lim_{x\to+\infty}(x+3)\left(\frac{x+1}{x+3} - \frac{\ln(x+3)}{(x+3)}\right)$$

$$= +\infty$$

لأن: « $\lim_{x\to+\infty} \frac{x+1}{x+3} = 1$ ; $\lim_{x\to+\infty} x+3=+\infty$

و $\lim_{x\to+\infty} \frac{\ln(x+3)}{x+3} = 0$ »

حساب: $\lim_{x\to-3^+} f(x)$

$$\lim_{x\to-3^+} x+1-\ln(x+3) = +\infty$$

لأن « $\lim_{x\to-3^+} \ln(x+3) = -\infty$

$\lim_{x\to-3^+} x+1 = -2$ »

②

$$\lim_{x\to+\infty} f(x) = +\infty$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} 1+\frac{1}{x} - \frac{\ln(x+3)}{x}$$

$$= \lim_{x\to+\infty} 1+\frac{1}{x} - \frac{\ln\left(x\left(1+\frac{3}{x}\right)\right)}{x}$$

$$= \lim_{x\to+\infty} 1+\frac{1}{x} - \frac{\ln(x)}{x} - \frac{\ln\left(1+\frac{3}{x}\right)}{x}$$

$$= 1$$

لأن: « $\lim_{x\to+\infty} \frac{\ln(x)}{x} = 0$

و $\lim_{x\to+\infty} \frac{\ln\left(1+\frac{3}{x}\right)}{x} = 0$ »

$$\lim_{x\to+\infty} f(x)-x = \lim_{x\to+\infty} 1-\ln(x+3)$$

$$= -\infty$$

« $\lim_{x\to+\infty} \ln(x+3) = +\infty$ »

إذن $C_f$ يقبل فرعا شلجميا باتجاه المستقيم : $y=x$

③

$$Z_1\times Z_2 = \left[2\sqrt{2}\times\sqrt{3} ; \frac{5\pi}{6} - \frac{\pi}{4}\right]$$

$$= \left[2\sqrt{6} ; \frac{7\pi}{12}\right]$$

- تحديد الشكل الجبري لـ $Z_1\times Z_2$ و $\cos\frac{7\pi}{12}$

$$Z_1\times Z_2 = \left(-\frac{3}{2} + \frac{\sqrt{3}}{2}i\right)\left(2-2i\right)$$

$$= -3+\sqrt{3} + i\left(6+\sqrt{3}\right)$$

$$\cos\frac{7\pi}{12} = \frac{\sqrt{3}-3}{2\sqrt{6}}$$

$$= \frac{\sqrt{6}\left(\sqrt{3}-3\right)}{12}$$

$$= \frac{3\sqrt{2}-3\sqrt{6}}{12}$$

$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\sin\frac{7\pi}{12} = \frac{6+\sqrt{3}}{2\sqrt{6}}$$

$$= \frac{\sqrt{6}\left(6+\sqrt{3}\right)}{12}$$

$$= \frac{6\sqrt{6}+3\sqrt{2}}{12}$$

$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

التمرين 04:

الجزء 1:

+ تحديد $D_f$:

$$D_f = \{x\in\mathbb{R} \;/\; x+3>0\}$$

$$D_f = \{x\in\mathbb{R} \;/\; x>-3\}$$

$$D_f = ]-3;+\infty[$$

3 - أ - حساب المشتقة:

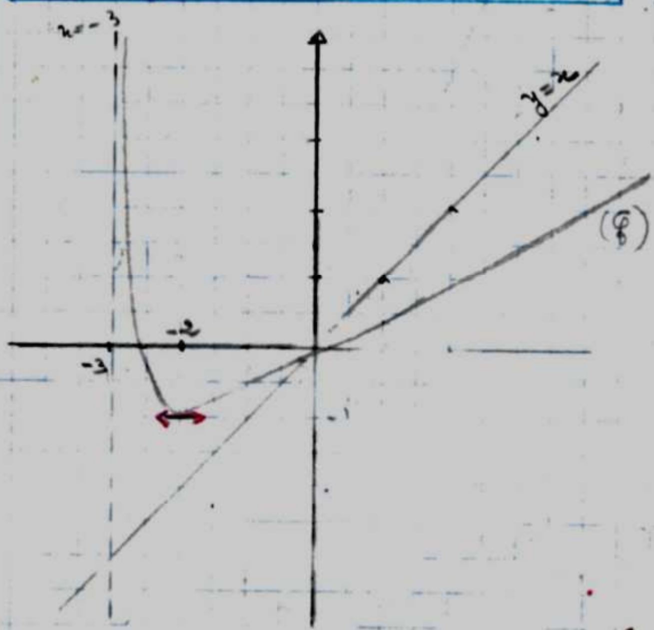
$$f'(x) = 1' - \frac{(x+3)'}{x+3} + x'$$

$$f'(x) = -\frac{1}{x+3} + 1$$

$$f'(x) = \frac{x+3-1}{x+3}$$

$$f'(x) = \frac{x+2}{x+3}$$

| $x$ | $-3$ | $-2$ | $+\infty$ |
|---|---|---|---|
| $f'(x)$ | $-$ | $0$ | $+$ |
| $f(x)$ | $+\infty$ ↘ | $-1$ | ↗ $+\infty$ |




① الجزء 2:

1 - $u_1 = f(u_0) \Leftrightarrow u_1 = 2+1-\ln(5)$ -1

$u_1 = 3 - \ln(5)$

لدينا $e < 5 \Leftrightarrow \ln e < \ln 5$

$\Leftrightarrow -1 > -\ln 5$

$\Leftrightarrow 2 > 3 - \ln 5$

$\Leftrightarrow u_0 > u_1$

2- نبين أن $\forall n \in \mathbb{N}\ -2 \leq u_n \leq 2$

من أجل $n=0$ : $-2 \leq u_0 = 2 \leq 2$ (V)

نفترض أن $-2 \leq u_n \leq 2$

ونبين أن $-2 \leq u_{n+1} \leq 2$ .

لدينا $-2 \leq u_n \leq 2$ و $f$ تزايدية

على المجال $[-2,2] \subset [-2;+\infty[$

إذن $f(-2) \leq f(u_n) \leq f(2)$

$f(2) = 3 - \ln 5 < 2$

$-2 \leq -1 \leq u_{n+1} \leq 2$

$-2 \leq u_{n+1} \leq 2$

ومنه $\forall n \in \mathbb{N}\ -2 \leq u_n \leq 2$

3- رتابة $u_n$:

لدينا $u_0 > u_1$

نفترض أن $u_n > u_{n+1}$

ولنبين أن $u_{n+1} > u_{n+2}$

لدينا $u_n > u_{n+1}$

و $f$ تزايدية على المجال $[-2,2]$

$f(u_n) > f(u_{n+1})$

$u_{n+1} > u_{n+2}$

إذن $\forall n \in \mathbb{N}\ u_n > u_{n+1}$

وعليه فإن $u_n$ تناقصية.

4- لدينا $u_n$ تناقصية ومحدودة بـ $-2$

إذن متقاربة

* $f$ مستمرة و $u_0 \in [-2,2]$

$f([-2,2]) = [-2;\ 3-\ln(5)] \subset [-2,2]$

إذن نهاية $u_n$ هي حل للمعادلة $f(x)=x$

$f(x) = x \Leftrightarrow 1 - \ln(x+3) = 0$

$\ln(x+3) = \ln e$

$x = e - 3.$

$\lim_{n\to+\infty} u_n = e - 3$